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Does Fiscal Policy Really Matter? Estimating the Optimal Government Size

Nimra Zulfiqar	Faculty of Business and Economics, University of Narowal, Narowal, Pakistan nimrazulfiqarhrx@gmail.com
Faiza Boota **(Corresponding Author)**	Faculty of Business and Economics, University of Narowal, Narowal, Pakistan faizakomal51214@gmail.com
Muhammad Ashraf	Faculty of Business and Economics, University of Narowal, Narowal, Pakistan muhammad.ashraf@uon.edu.pk
Muhammad Shahid Anwar	Faculty of Business and Economics, University of Narowal, Narowal, Pakistan shahidmayo832@gmail.com

Abstract: *Pakistan, a developing nation grappling with significant economic challenges, faces persistent issues related to budgetary deficits and escalating external debt. This study explores and estimates the optimal size of government and taxation system in Pakistan by employing the Ordinary Least Squares (OLS) method on annual data from 2000 to 2024. After performing diagnostic tests to ensure stationarity, the study utilizes the Engle-Granger cointegration analysis to uncover the long-term relationships between key economic variables. The Scully model is applied to determine the optimal size of government under both balanced and unbalanced budget scenarios, offering valuable insights into fiscal policy alignment. The results show that the short-run optimal size of government is 18.87%, while the long-run optimal size, derived through cointegration analysis, is 16.71%. Additionally, the study finds that the optimal level of government spending is 22.18%, while the optimal tax rate in an unbalanced budget scenario is 16.27(1-θ)%, with θ representing the fiscal deficit. The findings reveal a widening gap between the optimal taxation level and actual tax receipts, suggesting the need for Pakistan to shift from an indirect tax structure to a more direct tax system. Furthermore, expanding the tax net to include untaxed sectors and implementing stronger enforcement measures to reduce tax evasion and avoidance are crucial to improving tax compliance. Failure to achieve an optimal government size could lead to inefficient public spending, increased fiscal deficits, and underperformance in key sectors of the economy, ultimately hindering sustainable economic growth and development.*

Keywords: Optimal government size; Balanced budget; Unbalanced budget; Scully model; Taxation system

Introduction

Economic growth has long been a focus of research due to its potential to improve global living standards. Countries with sustained growth, such as Japan and China, have witnessed substantial advancements in infrastructure, healthcare, and education, significantly improving the quality of life for their citizens. These examples highlight the transformative potential of economic growth. Sustained growth, driven by resource allocation, sound economic policies, and private sector investment, is crucial to improving societal well-being. Governments play a vital role in economic growth through taxation and public spending, which influence individual decisions on saving, investment, and resource allocation. Government revenue is essential for providing public goods and fostering a conducive environment for growth. In this context, determining the optimal size of government spending and taxation is essential for balancing economic growth with equity. The debate surrounding the optimal size of government centers on the balance between fostering economic growth and ensuring equity. The BARS curve (Barro, Armey, Rahn, and Scully), which connects the size of government to economic growth, suggests an optimal government size beyond which growth declines. The curve has been widely studied, employing different econometric techniques based on available data, including time series, cross-sectional, and panel data.

Public spending is a key factor in determining the success of government policies. Effective spending enhances a government's ability to provide essential services, support social safety nets, and reduce inequality. However, insufficient resources can hamper economic progress and governance. Thus, optimizing government spending and taxation is critical to achieving growth while maintaining fiscal discipline. Previous studies have analyzed the BARS curve, demonstrating varying optimal levels of government taxation and expenditure. For example, Chao and Grubel (1998) estimated the optimal taxation level in Canada at 34% of GDP, while Vedder and Gallaway (1998) explored the impact of government size on growth in the US and Europe. Scully (2003) refined this analysis by incorporating income distribution, concluding that the optimal tax rate for the US was 19.3% of GDP. These studies underscore the importance of balancing government size to achieve sustainable growth.

In developing countries like Pakistan, the optimal government size and taxation policies face unique challenges. Pakistan's low tax-to-GDP ratio, high reliance on indirect taxes, and structural fiscal deficits create significant barriers to optimizing government spending and taxation. Additionally, issues such as economic informality and limited institutional capacity complicate the development of effective fiscal policies. This study applies the Scully model to estimate the optimal government size in Pakistan, focusing on both balanced and unbalanced budget scenarios. A balanced budget, where government expenditure equals revenue, is crucial for maintaining fiscal discipline, while unbalanced budgets often arise due to factors such as low tax collection and heavy reliance on external borrowing. Understanding these dynamics is vital to aligning fiscal policies with Pakistan's growth objectives. The objective of this study is to determine the optimal level of taxation and expenditure in Pakistan using the BARS curve framework. This research will provide valuable insights into how the government can optimize its role in fostering sustainable economic growth, improving living standards, and achieving sustainable development. Given the unique challenges faced by Pakistan, this study aims to fill a gap in the existing literature, offering recommendations tailored to Pakistan's economic context.

Many studies have analyzed the optimal size of government in developed countries, few have focused on developing economies like Pakistan. This study aims to fill that gap, providing evidence-based recommendations for policymakers to enhance Pakistan's economic growth and living standards. By applying the Scully model, this research contributes to the broader discourse on government and economic growth, offering practical insights for academics and practitioners alike.

The structure of the paper is organized as follows: Section II provides a concise review of the relevant literature. Section III outlines the methodology used and describes the data employed in our study. Section IV presents the key findings derived from the research. Finally, Section V offers conclusions and discusses policy recommendations.

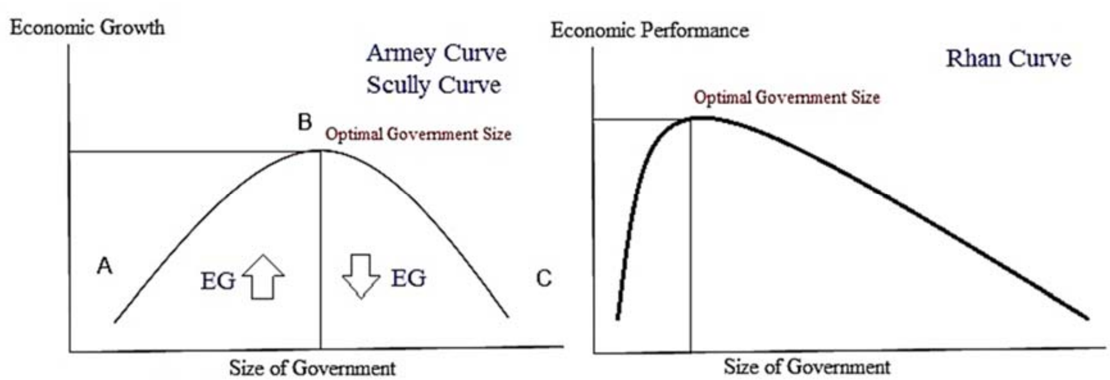
2. Literature Review

2.1. Theoretical Foundations

The relationship between the optimal size of government and economic growth is illustrated by the BARS curve, named after the works of Barro (1990), Armey (1995), Rahn (1996), and Scully. This curve forms the theoretical foundation for the literature on the optimal size of government and its impact on economic performance.

In 1990, Robert J. Barro emphasized the dual impact of public sector spending on economic growth. On one hand, an increase in government spending typically leads to higher taxation, which can negatively affect economic growth by reducing private sector activity. On the other hand, increased public expenditure can enhance economic growth by boosting the marginal productivity of resources. Barro argued that if the government size is large, the negative effects of high taxation tend to dominate, thereby inhibiting growth. However, in smaller governments, the positive effects of increased spending are more likely to outweigh the negative impacts of higher taxes, leading to enhanced growth. Later on, in 1995, Richard D. Armey extended this theory by proposing that as government size increases, GDP initially rises, but beyond a certain point, further increases in government size result in diminishing returns to economic growth. This highlights the existence of an optimal government size, after which the relationship between government size and growth turns negative. Armey's framework supports the notion of a positive relationship between government size and economic growth up to a critical threshold, beyond which the effect becomes detrimental.

Rahn et al. (1996) further developed this understanding, particularly in the context of the United States. They argued that the extensive government spending in the U.S. negatively impacted economic growth, as the government was less productive and efficient compared to the private sector. Additionally, high taxation burdened the economy, stifling growth. This underscores the importance of optimizing government size to ensure efficiency and prevent the economy from being weighed down by excessive taxation. Scully (1994) also has the important contributor to the understanding of the significant role of government size in economic growth. He observed a significant decline in U.S. productivity during the 1950s, attributing much of this decline to an oversized government. While policymakers often point to factors such as inadequate physical and human capital, insufficient research and development, and energy crises as the primary causes of stagnation, Scully argued that the root cause was the government's disproportionate size. He highlighted that an oversized government drains resources from the private sector, hindering economic growth. To foster optimal economic growth, Scully suggested that the optimal level of government taxation should be between 22% and 23% of total GNP.



2.2. Empirical Evidence

Several studies have empirically examined the relationship between the optimal size of government and economic growth, employing various frameworks and data sets. Chao and Grubel (1998) analyzed the impact of the optimal government size on the Canadian economy between 1929 and 1996. Assuming a balanced budget throughout the period, they concluded that the optimal taxation level for Canada was approximately 34%, while the optimal government size was around 25% of total GDP. This result provides valuable insights into the fiscal dynamics of developed economies. Similarly, Chobanov and Mladenova (2009) argue that the optimal government size should generally not exceed 25% of GDP, underscoring a threshold for efficient public sector participation.

Vedder and Gallaway (1998) utilized the Armey curve to explore the optimal taxation and expenditure levels in both developed and developing economies. They found that in the United States and Europe, slow economic growth coincided with a substantial increase in government size relative to GDP. In contrast, growth in the private sector was more pronounced in several Asian economies, which were positioned on the upward-sloping portion of the Armey curve. This positioning indicates that these economies were able to benefit from government spending up to a certain point, beyond which further expansion of government size could become detrimental. The study's findings were supported by their analysis of U.S. data from 1947 to 1997, where they estimated the optimal government size at approximately 17.45% of GDP.

Scully (2003) further refined this concept by developing a model that estimates the optimal size of taxation and government expenditure in the U.S. economy. Using data from 1960 to 1990, he concluded that the optimal tax rate for the U.S. economy is around 19.3% of GDP, a level that he identified as maximizing economic growth. His findings highlight the significant role that tax policy plays in fostering an environment conducive to growth. Afonso et al. (2005) examined public sector performance and efficiency in industrialized nations, concluding that a negative correlation exists between the size of the public sector and its efficiency. Their study found that EU-15 countries were generally less efficient than OECD countries and the U.S. Furthermore, Afonso and Furceri (2010) suggested that a 1% increase in public spending could reduce growth by 0.12% in OECD countries and 0.13% in EU countries. These findings emphasize the importance of public sector efficiency in shaping the growth outcomes of a nation.

Grossman (1988) proposed a nonlinear relationship between government size and economic growth using U.S. time series data. His findings indicate that beyond a certain threshold, any increase in government size inversely affects economic growth, reinforcing the argument for an optimal government size. In a similar vein, Chobanov and Mladenova (2009) also argue that the optimal government size should not exceed 25% of GDP. Forte and Magazzino (2010) extended the BARS curve analysis to the European Union, finding that the curve suggests an optimal expenditure share of 37% of total GDP, whereas the actual average expenditure share was 47%. Their study revealed variations across EU countries, with Ireland (44.47%), the UK (43.50%), and Portugal (42.28%) exhibiting higher government spending levels, while Belgium (35.39%), the Netherlands (35.52%), and Italy (37.68%) were at the lower end of the spectrum. These differences highlight the diverse fiscal environments within the EU and underscore the complexity of determining an optimal government size across different contexts.

Karagianni and Pempetzoglou (2011) further examined the relationship between national income and public spending, utilizing different functional forms of Wagner's Law. Their findings suggest that national income and public spending face significant challenges in terms of nonlinearity, making it difficult for policymakers to precisely determine the optimal government size, especially in response to economic shocks. Similarly, Afonso and Jalles (2011) developed a growth model to investigate the relationship between private consumption, public spending, and output per labor across more than 100 countries. Their results indicate that government size can have an inverse effect on growth, particularly in countries with weak legal systems, poor governance, and inadequate political rights. Conversely, they found that institutional quality positively affects economic growth, emphasizing the importance of good governance in determining the effectiveness of government spending.

Makin et al. (2019) explored this gap by analyzing the factors behind the growth of government size and its broader implications for the economy. They argue that there exists a trade-off between increased government size and economic growth, with diminishing returns beyond an optimal threshold, as represented by the BARS curve. Focusing on Australia, the authors found that, post-GFC, government spending had reached a historically high level of 37% of national income. Through econometric estimation using the ARMAX approach, the study concluded that the optimal government spending share for maximizing economic growth in Australia is approximately 31% of national income, significantly below the current level.

Similarly, Akram and Rath (2020) conducted an analysis on Indian states from 1990-91 to 2017-18, using panel threshold regression models to examine the relationship between optimal government size and economic growth. Their findings revealed that government size positively and significantly influenced economic growth within a certain threshold. However, once the government size surpassed the upper threshold, its impact began to diminish and eventually became insignificant. This suggests that policymakers should aim to keep government size within these optimal thresholds for sustaining economic growth.

Buljan et al. (2022) examined the relationship between local government size and economies of scale in Croatia, finding a statistically significant U-shaped relationship between per capita local expenditures and population size. This relationship was analyzed while controlling for various demographic, socio-economic, and institutional factors. The results showed that the population size of most cities and municipalities in Croatia was below the optimal level, with 72% of cities and 76% of municipalities operating below the optimal population size based on median estimates. This study provides valuable insights into how local government size can be optimized for efficiency and cost-effectiveness.

Rodzinka et al. (2022) analyzed the optimal size of the general government sector in the EU by employing the innovative decision tree technique (C5.0 method). The study covered data from 28 EU member states between 2000 and 2017, analyzing 16,632 input data points. Their findings highlighted that there is no universally optimal size for the general government sector, but several dependencies between variables can be identified. The study suggests that, by understanding the impact of specific government actions on the economy, policymakers can design strategies that optimize government size without adversely affecting other sectors.

Abdillah (2023) investigated the long-term and short-term relationship between government size and economic growth in Indonesia using the ARDL model and secondary data from the World Development Indicators for the period 1966-2021. The study found that, in the short run, government size had a significant positive effect on economic growth. However, when examining the relationship in the long term with a quadratic form of the government size variable, the study revealed a negative relationship beyond a certain threshold. The safe limit for government expenditure was identified at 57.9% of national income, supporting the Armey curve hypothesis. The study recommended that policymakers control government spending to avoid surpassing this optimal threshold, ensuring sustainable economic growth.

2.3. Conceptual framework

The estimation of the optimal size of government has been extensively studied in the literature, with the Scully model widely regarded as a standard approach. However, this model assumes a balanced budget, where government expenditure equals tax receipts. This assumption does not align with the fiscal realities of most nations, where fiscal deficits are a common occurrence. Consequently, this assumption will be relaxed in the extended model discussed below.

2.3.1. Balance budget:

The Scully model, based on a non-linear Cobb-Douglas production function, is mathematically specified as $Y = \delta \left(\frac{G}{Y}\right)^{\alpha} + (1-\tau)^{\beta}$. Where Y and G represent the total output and government expenditure and, α & β represent the relative share of government and private sector. However, τ indicates the Tax to GDP ratio and δ indicates the total factor productivity. Taking the natural logarithm of both sides of the equation yields $\ln Y = \ln \delta + \alpha \ln \left(\frac{G}{Y}\right) + \beta \ln(1-\tau)$. To assess the impact of government expenditure on economic growth, we differentiate the model with respect to G which is $\frac{\partial \ln Y}{\partial G} = \alpha\, G^{-1}$ and second differentiating with respect to G is $\frac{\partial^2 \ln Y}{\partial G^2} = -\alpha\, G^{-2}$. Since $-\alpha\, G^{-2} < 0$ indicates that while public expenditure positively impacts economic growth up to a certain point, beyond that point, it begins to have an adverse effect on the economy.

Under the assumption of a balanced budget, where i.e. G = T. substituting this into the equation results in $\ln Y = \ln \delta + \alpha \ln(\tau) + \beta \ln(1-\tau)$. To determine the optimal size of government, we take the first derivative with respect to τ. Hence, optimal size is $\tau = \frac{\alpha}{\alpha+\beta}$ This equation implies that the optimal tax rate, which maximizes economic growth under a balanced budget, is the ratio of the relative share of the public sector α to the total share of both the public and private sectors $(\alpha + \beta)$.

2.3.2. Unbalance Budget:

Recognizing the fiscal realities of most developing economies, where fiscal deficits are prevalent, Husnain et al. (2015) extended the Scully model to accommodate unbalanced budgets. This extension is more relevant for developing countries, where fiscal deficits are a common feature. The model specification in this case builds on the Cobb-Douglas production function, modified to account for both public and private sector contributions, as well as fiscal deficit: $Y_t = A\,(\tau Y_{t-1})^{\alpha}\,(\theta Y_{t-1})^{\beta}\,\left((1-\tau-\theta)Y_{t-1}\right)^{\gamma}$ whereas, Y shows the total GDP of a nation at current period (t) and previous year (t-1) and A shoes the total productivity. Relative share of government, deficit and private sector shows by $(\alpha, \beta\ and\ \gamma)$ and $(\tau, \theta\ and\ (1-\tau-\theta))$ is shows the tax to GDP ratio, deficit to GDP ratio and share of private sector. As we know $g = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$, and after rearranging it will be $1 + g = \frac{Y_t}{Y_{t-1}}$. Substituting value of Y_t and talking natural log then model will be $\ln(1+g) = \ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln A + \alpha \ln \tau + \beta \ln \theta + \gamma \ln(1-\tau-\theta)$. In order to find the optimal tax rate for unbalance budget lets apply first order condition and solve for τ and results will be $\tau = \frac{a(1-\theta)}{(\alpha+\gamma)}$. Where, θ shows the fiscal deficit and $\frac{a}{(\alpha+\gamma)}$ is the reduced form parameter which implies the share of government tax in total production to the sum of total government tax and share of private sector.

3. Econometric techniques

The study used the secondary time series dataset includes variables such as gross domestic product (GDP), taxation as a percentage of GDP, public spending as a percentage of GDP, the growth rate, and fiscal deficit as a percentage of GDP, covering the period from 2000 to 2024. The data is sourced from the State Bank of Pakistan (SBP). Some variables are constructed using the aforementioned data points, and to ensure consistency in the regression analysis, a log transformation is applied to all variables.

The study employs time-series data to estimate the optimal tax rate. Time-series data is often non-stationary, meaning that the series exhibit trends over time. To assess stationarity, the Augmented Dickey-Fuller (ADF) unit root test is applied. Non-stationary series can lead to spurious regression results, rendering estimated coefficients unreliable. Therefore, to ensure stationarity, the first difference of the series is taken, effectively removing the trend. The Ordinary Least Squares (OLS) method is used as the primary technique for estimating the regression coefficients for both balanced and unbalanced budget scenarios. The estimated coefficients derived from these models are then utilized to calculate the optimal size of government for the respective economies.

$$\Delta Y_t = \alpha_t + \sum_{i=1}^{n} \beta_i\, \Delta Y_{t-i} + \sum_{i=1}^{m} \beta_i\, \Delta X_{t-i} + \varepsilon_t$$

Additionally, to capture long-run relationships among the variables, cointegration analysis i.e. Engel Granger cointegration is conducted. This analysis allows us to identify any equilibrium relationships between key economic indicators, such as GDP, taxation and public spending as a percentage of GDP, fiscal deficit, and growth rate.

$$\Delta u_t = \varrho u_{t-1} + \sum_{i=1}^{\rho} \delta_i \, \Delta u_{t-i} + \varepsilon_t$$

4. Empirical Results and Discussion:

This section presents the results of the Augmented Dickey-Fuller (ADF) test, which assesses the stationarity of the variables used in the analysis. Additionally, it outlines the estimation of the optimal size of taxation and government spending under both balanced and unbalanced budget scenarios. The results provide insights into the optimal tax rate and government expenditure required to maximize economic growth, addressing the key issues of fiscal deficits and budgetary imbalances. The findings also emphasize the implications of achieving or failing to achieve the optimal government size, which significantly impacts Pakistan's economic performance and long-term sustainability.

4.1. Result of Augmented Dickey-Fuller Test:

Table 1 presents the results of the Augmented Dickey-Fuller (ADF) test. The null hypothesis of the ADF test posits that the series has a unit root (i.e., it is non-stationary), while the alternative hypothesis suggests that the series is stationary. The results indicate that only one variable, growth, is stationary at the level, while the remaining variables are stationary after taking the first difference. Given that the coefficients are used to determine the optimal size of government in both balanced and unbalanced budget settings, Ordinary Least Squares (OLS) is deemed the most appropriate technique for estimating the regression coefficients. This approach ensures reliable estimation of the relationships between the variables once stationarity is achieved.

Table No. 4.1: ADF Unit Root Test

Variables	Deterministic Part	Lag	Level			1st Difference	
			ADF	P-Value	Deterministic Part	ADF	P-Value
LnY	C&T	0	-1.990	0.286	C	-8.126	0.000
LnT	C	0	-2.017	0.235	None	-7.559	0.000
Ln(1-T)	C	0	-2.457	0.233	None	-7.861	0.000
LnS	C&T	0	-2.987	0.486	None	-8.124	0.000
Ln(1- ɛ)	C&T	0	-2.199	0.487	None	-6.245	0.000
Lnθ	C	0	-2.277	0.109	None	-7.031	0.000
Ln(1-τ-θ)	C&T	0	-2.158	0.441	None	-8.799	0.000
Ln(1+g)	C	0	-5.136	0.003	None	-11.194	0.000

Source: Author's Own Calculation

4.2. Result of Optimal Size of Taxation in balance budget:

It is assumed that the country operates under a balanced budget, where taxation equals government spending. Based on this assumption, two separate estimations were conducted: one to determine the optimal level of taxation and the other to estimate the optimal level of government spending. After ensuring stationarity, the OLS estimation results are presented in Table 4.2.

Table No. 4.2: Regression Result With and Without Trend

Dependent variable: Δ Ln Y			
Without Trend			
Variables	**Coefficient**	**Standard Error**	**P-Value**
C	0.135	0.005	0.000
Δ Ln τ	-1.592	0.589	0.000
Δ Ln (1-τ)	-6.842	3.254	0.000
R Square	0.428		

Durbin Watson	1.887		
Note: Δ denoted first difference operator			
With Trend			
Variables	**Coefficient**	**Standard Error**	**P-Value**
C	6.108	2.810	0.000
Ln τ	-2.109	1.008	0.000
Ln (1-τ)	-10.51	5.512	0.000
@Trend	0.135	0.001	0.000
R Square	0.799		
DW	1.987		

The optimal level is calculated as $\tau = \frac{\alpha}{\alpha+\beta} = \frac{-1.59}{-1.59\ -6.84}$ and yielding an optimal level of 18.87%. This optimal size reflects the short-run result. To further investigate the long-run relationship, the Engle-Granger two-step method was employed to test for cointegration.

Table No. 4.3 (a): Engel Granger Test (First Step)

Test Statistics	**t-Statistic**	**Prob.***
Augmented Dickey-Fuller	-4.349	0.001

Table No. 4.3 (b): Engel Granger Test (Second Step)

Variables	**Coefficient**	**Standard Error**	**P-Value**
C	0.135	0.004	0.000
Ln τ	-1.663	0.027	0.000
Ln (1-τ)	-7.347	1.085	0.001
EC(-1)	-0.271	0.012	0.000
R Square	0.595		
Durbin Watson	1.917		

Table 4.3 (a) presents the results from the first step of the Engle-Granger method. In this step, the ADF test is calculated which is significant; indicate that error term is stationery. This implies that cointegration is present. Table 4.3 (b) results yield the results of Engle-Granger second step. In, Engle-Granger step two, optimal size $\tau = \frac{\alpha}{\alpha+\beta} = \frac{-1.66}{-1.66\ -7.34}$ is 18.46%.

Therefore, a comparison of optimal level of taxation is calculated by different method and results are same i.e. OLS estimation and Engle-Granger two step method. By comparing different method possibility of spurious regression also ruled out. In both cases, optimal level of taxation is above 18%.

4.4. Result of Optimal Size of Spending in Balance Budget:

It is assumed that the country maintains a balanced budget, meaning taxation equals government spending. The results presented in Table 4.1 indicate that all variables became stationary after the first difference was applied. Consequently, Ordinary Least Squares (OLS) estimation was used to analyze the data.

Table No. 4.4: Regression Result With and Without Trend

Dependent variable: Δ Ln Y			
Without Trend			
Variables	**Coefficient**	**Standard Error**	**P-Value**
C	2.132	0.007	0.000
Δ Ln ɜ	1.714	0.057	0.000
Δ Ln (1-ɜ)	6.019	1.971	0.000
R Square	0.721		
Durbin Watson	1.756		

Note: Δ denoted first difference operator			
With Trend			
Variables	**Coefficient**	**Standard Error**	**T-Value**
C	1.136	0.006	0.000
Ln ƨ	1.941	0.685	0.000
Ln (1- ƨ)	6.912	1.521	0.000
@Trend	-0.405	0.006	0.000
R Square	0.271		
Durbin Watson	1.967		

Table 4.4 presents the OLS estimation results, with and without a trend. The optimal level of spending is derived as $\tau = \frac{\alpha}{\alpha + \beta} = \frac{1.71}{1.71 + 6.01}$, yielding an optimal spending level of 22.15%. Since all variables were stationarized by taking their first differences, the model primarily captures short-run dynamics. To incorporate both short-run and long-run properties, the Engle-Granger estimation is utilized, as it effectively handles variables measured at their levels.

Table No. 4.5 (a): Engel Granger Test (First Step)

Test Statistics	**t-Statistic**	**Prob.***
Augmented Dickey-Fuller	-4.607	0.000

Table No. 4.5 (b): Engel Granger Test (Second Step)

Variables	**Coefficient**	**Standard Error**	**P-Value**
C	17.17999	1.633	0.000
Ln ƨ	2.231526	0.673	0.000
Ln (1- ƨ)	7.967152	1.443	0.001
EC(-1)	0.137442	0.0001	0.000
R Square	0.695		
Durbin Watson	0.026		

Table 4.5 (a) presents the results of the first step of the Engle-Granger procedure. The ADF test value is significant, indicating that the error term is stationary, thereby confirming the presence of cointegration among the variables. Table 5(b) provides the results of the second step, where the optimal spending level is calculated as $= \frac{\alpha}{\alpha + \beta} = \frac{2.23}{2.23 + 7.96}$, yielding an optimal spending level of 21.88%.

A comparison of the optimal taxation levels derived through different methods i.e. OLS estimation and the Engle-Granger two-step procedure, reveals consistent results, with both approaches yielding optimal taxation levels above 18%. This alignment across methodologies further rules out the possibility of spurious regression and strengthens the robustness of the findings.

4.6. Result of Optimal Size of Spending in Unbalance Budget:

For the unbalanced budget framework, the estimated equation is expressed as $\ln(1+g) = \ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln A + \alpha \ln \tau + \beta \ln \theta + \gamma \ln(1 - \tau - \theta)$. The optimal government expenditure is derived as $\tau = \frac{\alpha(1-\theta)}{(\alpha+\gamma)}$ where θ denotes the fiscal deficit. The OLS estimation results for the unbalanced budget, presented in Table 6, indicate an optimal taxation level of $\tau = \frac{0.057(1-\theta)}{(0.057+0.289)}$ which equates to 16.27(1-θ)%. These findings highlight the dependency of the optimal tax rate on the fiscal deficit, reflecting the nuanced interplay between taxation and fiscal imbalances.

Table No. 4.6: OLS Regression Result for Unbalance Budget Approach

Variables	Coefficient	Standard Error	P-Value
C	0.047	0.003	0.000
Ln τ	0.057	0.006	0.000
Ln θ	0.004	0.003	0.001
Ln $(1-\tau-\theta)$	0.289	0.000	0.000
R Square	0.715		
Durbin Watson	1.715		

Various diagnostic tests were conducted to evaluate potential issues such as model specification errors, autocorrelation, heteroskedasticity, stability, and normality. The Ramsey RESET F-test was employed to assess specification errors in the model. To verify the normality of the residuals, the Jarque-Bera test was performed. The Breusch-Godfrey LM F-test was used to detect serial correlation, while the ARCH F-test examined the presence of autoregressive conditional heteroskedasticity. Additionally, the CUSUM test was applied to evaluate the stability of the model over time. Lastly, summary of the results are shown in the table 4.7.

Table # 4.7: Summary of the Results

No.	Approach	Variable	Technique	Results
1	Balance Budget	Taxation	OLS	18.87%
			Engel Granger	18.46%
2	Balance Budget	Public Spending	OLS	22.18%
			Engel Granger	21.93%
3	Unbalance Budget	Taxation	OLS	16.27(1-θ) %

5. Conclusion, policy implications and future directions:

5.1. Conclusion

Pakistan, as a developing nation with a rapidly growing population, grapples with a multitude of economic challenges, including a persistent budget deficit and escalating external debt. This study employed the Ordinary Least Squares (OLS) method using annual data from 2000 to 2022, supplemented by cointegration analysis to explore long-term relationships among critical economic variables. The findings underscore a fundamental issue: the inability to align expenditures with available resources, resulting in significant economic strain.

Addressing this challenge requires two fundamental approaches, either increasing resource generation or adopting fiscal discipline to operate within existing means. While the latter is often more challenging to implement, enhancing resource mobilization emerges as a primary solution. A key area of focus in this regard is Pakistan's low tax-to-GDP ratio, one of the lowest globally. The analysis reveals a substantial and widening gap between the optimal level of taxation and actual tax revenues, emphasizing the urgent need for reform.

To bridge this gap, Pakistan must expand its tax base to include currently untaxed sectors and address inefficiencies within the tax system. Furthermore, the country's reliance on indirect taxation undermines the potential for sustainable growth in tax revenues. A shift toward a more balanced and progressive taxation system is critical for enhancing revenue generation and achieving fiscal stability. These measures are essential for Pakistan to mitigate its economic challenges and pave the way for sustainable growth..

5.2. Policy Implications

To enhance Pakistan's tax-to-GDP ratio and address fiscal imbalances, a comprehensive approach is required. A critical first step is transitioning from a reliance on indirect taxation to a more equitable and progressive direct tax system. This shift would ensure a fairer distribution of the tax burden and create a sustainable foundation for revenue generation. Additionally, expanding the tax base is imperative. Untaxed and under-taxed sectors must be brought within the tax net to promote broader compliance and alleviate the pressure on existing taxpayers.

Strengthening the legal framework is another vital measure. Enforcing stringent laws to deter tax evasion and avoidance, along with imposing strict penalties, will enhance accountability and reduce revenue leakages. Moreover, tax reforms are necessary to simplify procedures and modernize the taxation system. Leveraging technology to streamline tax collection processes will not only improve efficiency but also encourage voluntary compliance by reducing bureaucratic obstacles.

Finally, it is crucial to align revenue generation targets with the country's fiscal requirements. Tax policies should account for the budgetary deficit to ensure that revenues meet fiscal needs without compromising economic stability. By implementing these measures, Pakistan can significantly improve its tax-to-GDP ratio, strengthen fiscal stability, and foster sustainable economic growth.

5.3. Future Directions

Future research could delve deeper into the structural factors underlying Pakistan's low tax-to-GDP ratio, focusing on the social, cultural, and institutional barriers that impede tax compliance and collection. Investigating the potential transition from an indirect to a direct tax system is also essential. Comparative case studies of countries that have successfully reformed their taxation systems could offer practical insights and lessons for Pakistan. Additionally, future studies should analyze the long-term effects of enhanced tax receipts on economic growth, external debt reduction, and budgetary deficit management to provide more robust and actionable policy recommendations.

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